

A Compositional Semantics for ‘If Then’ Conditionals[★]

Mathieu Vidal^{★★}

¹ Cognitions Humaine et Artificielle (CHArt), Université Paris 8

² Philosophie, Pratiques & Langages (PPL), Université Grenoble Alpes

Abstract. This paper presents the first compositional semantics for *if then* conditionals. The semantics of each element are first examined separately. The meaning of *if* is modeled according to a possible worlds semantics. The particle *then* is analyzed as an anaphoric word that places its focused element inside the context settled by a previous element. Their meanings are subsequently combined in order to provide a formal semantics of *if A then C* conditionals, which differs from the simple *if A, C* form. This semantics has the particularity of validating contraposition for the first type but invalidating it for the second type. Finally, a detailed examination of the sentences presented in the literature opposing this schema of reasoning shows that these counterexamples do not generally concern *if then* conditionals but, rather, *even if* conditionals and that contraposition is therefore a valid means of reasoning with regard to *if then* conditionals in natural language, as this system predicts.

Keywords: Conditional Logic, If, Then, Contraposition

Introduction

[16] has observed that the addition of *then* to *even if* conditionals is unsuitable:

- (1) Even if John is drunk, Bill will vote for him.
- (2) # Even if John is drunk, then Bill will vote for him.

This example clearly dispels the misconception whereby the addition of *then* to a conditional does not change its meaning. [30], [19], [1] and their followers have implicitly adopted this position in their theories, according to which no formal distinction is made between the *if A, C*, the *even if A, C*, and the *if A then C* forms. In this paper, I shall try to overcome this limitation by providing a compositional analysis of the meaning of *if then* conditionals. More precisely, I will present a formal semantics for both *if* and *then*, ultimately showing that their combination leads to a particular semantics for the *if then* form.

This article is the first formal attempt to provide a compositional analysis of the formal semantics of the *if then* conditional in terms of the elements *if* and *then*. Indeed,

[★] Last accepted manuscript version with some corrections. This article must be referenced as Mathieu Vidal, "A Compositional Semantics for ‘If Then’ Conditionals", *LACL 2016, LNCS 10054*, M. Amblard et al. (Eds.), pp. 291-307, 2016. The final publication is available at http://dx.doi.org/10.1007/978-3-662-53826-5_18.

^{★★} I would like to thank Philippe Schlenker for suggesting to develop the ideas from [37] in a compositional way, the referees and Denis Perrin for their help to improve the paper, and the editors for the publication of these proceedings.

previous explanations of the meaning of *then*, such as those proposed by [16] and [20] were pragmatic. On the contrary, I argue for a formal analysis because the mathematical description it employs provides a more precise explanation. Furthermore, its treatment can often be automated on a computer. Finally, by having a different semantics, the various conditionals validate different forms of reasoning.

This theory concerns both indicative and subjunctive conditionals. Indeed, a similar problem occurs in counterfactuals, such as in the following examples:

- (3) Even If John had been drunk, Bill would vote for him.
- (4) # Even If John had been drunk, then Bill would vote for him.

As the difference between the indicative and the subjunctive has no impact on this issue, I shall refrain from examining the moods used in conditionals in this paper. I will also not be dealing with embedded conditionals (as *if A then if B then C*) and chains of conditionals (as *if A then B and if B then C*). These forms of reasoning display dynamic features, so, for the sake of simplicity, I will limit my analysis to a static approach in this initial presentation of the theory.

This article is divided into four parts. Section 1 presents the semantics for *if*. Section 2 introduces the formal semantics for *then*, for cases both inside and outside conditionals. Section 3 deals with the compositional semantics obtained by combining *if* and *then* and details the main schemas of reasoning that are valid for this approach. Section 4 focuses on contraposition in order to show that its validity, which the theory predicts, dovetails the facts observed in natural language, contrary to what is usually argued in the field.

1 If

A conditional is a linguistic expression composed of two clauses. One clause, called the *protasis* or the *antecedent*, states the conditions under which the other clause, called the *apodosis* or the *consequent*, is considered true. The marking of conditional sentences differs greatly between natural languages. However, according to [5], the general strategy is to mark the protasis using a lexical element, a particular inflectional morphology, or a purely syntactic means. If we take English as our reference language, the simplest syntactic form of a conditional is *if A, C*, with *A* being the protasis, *C* the apodosis, and *if* the marker of conditionality.

For the semantics of the *if*, I will draw upon [38]’s proposal. The semantics presented in that paper were already successfully combined with the semantics of *even* in order to provide the meaning of the *even if* conditional. The other advantage of this approach is that it can be seen as a refined version of Stalnaker’s semantics, in which the problem of unconnected conditionals is resolved. For Stalnaker, a conditional *if A, C* is true in a possible world *w* if the closest *A*-world of *w* is also a *C*-world. [7] refined this semantics in order to select not only one world, but a set of possible worlds as the closest worlds in which the antecedent is true. That is the version I shall discuss here. Since *w* is the closest world to itself in this semantics, as soon as the antecedent is true in the initial world, this initial world is also the world in which the consequent is evaluated.³ The unfortunate result of this is that, if the consequent is also true in the ini-

³ According to Chellas’s semantics, the following condition holds: if $w \in [A]$, then $f_w(A) = \{w\}$.

tial world, the whole conditional is true, regardless of the links between the antecedent and the consequent. For instance, the sentence “if Mickey Mouse has four fingers per hand, Mickey Mouse has big ears” is automatically true, despite the lack of connection between its components. The associated schema of inference is called Conjunctive Sufficiency (CS): $A, C \models A \rightarrow C$.⁴ This inference of a conditional from a conjunction has already been criticized from an intuitive point of view by [22] and the psychological experiments conducted in [21] and [29] show that it is also not endorsed by ordinary subjects.

To resolve this issue, Vidal proposes breaking the process of evaluation down into two parts. Prior to this evaluation, the subject can hold one of three positions concerning the antecedent: it is either believed to be true, believed to be false, or believed to be neither true nor false. During the first step of evaluation, the antecedent is inhibited. This means that it is no longer believed to be true or false. The position concerning the antecedent is now neutral (neither true nor false). The first advantage of this inhibition is that, if the antecedent was previously believed to be false, it can now be added to the stock of beliefs without leading to a contradiction. The second advantage of this inhibition is that, if the antecedent was previously believed to be true, its addition can be now accomplished with slight variations. This addition to the stock of beliefs occurs during the second step of the evaluation, the antecedent being reconstructed in several different ways. If the consequent is obtained in all of the reconstructions of the antecedent, the conditional is declared true.

These ideas can be turned into a formal semantics. In the present paper, I will only expose the main elements. The reader is referred to Appendix A of [38] for all the technical details. A stock of beliefs is represented by a set of possible worlds. In our case and to put it simply, the initial beliefs are rendered by a unique possible world. As some sentences are no longer believed to be true or false during the judgment, some of the possible worlds of this semantics are trivalent. The step of inhibition is modeled through what is called a *neutralization function*, noticed n . The step of reconstruction is modeled through what is called an *expansion function*, noticed e . There are limits on the possible reconstructions of the antecedent. Some of them are too absurd or too improbable to be considered. Hence, only a part of them is envisaged, and they are constrained by what is called a *universe of projection*. An important aspect of this universe of projection is that, at the end of the judgment, the antecedent and the consequent are either true or false in each possible reconstruction envisaged.

This semantics for the conditional “if A , C ” is depicted in Fig. 1, in which w stands for the starting world of evaluation and the square for the universe of projection.

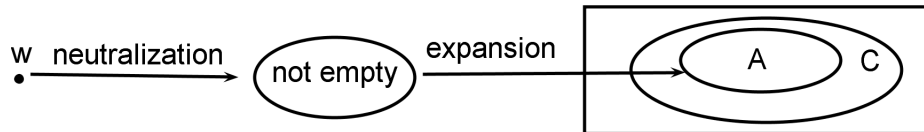


Fig. 1. Basic semantics of the conditional

⁴ The symbol \rightarrow stands for the *if* A , C conditional.

More formally, the meaning of the word *if* is the following:

Definition 1 (Meaning of If). *Let w be a possible world, λ the lambda abstractor, X and Y some sentences, $[X]$ and $[Y]$ their truth-sets, n a neutrality function and e an expansion function both governed by the universe of projection U used to evaluate the conditional at hand. Then, the **meaning of if** in a possible world w is*

$$[if]^w = \lambda X \lambda Y n_w(X) \neq \emptyset \wedge e_{n_w(X)}(X) \subseteq [Y]$$

We obtain the following truth-conditions for the sentence *if* A , C .

Theorem 1 (Truth-Conditions of If).

$\models_w (if A) C$ iff in the associated universe of projection U

- i) $n_w(A) \neq \emptyset$
- ii) $e_{n_w(A)}(A) \subseteq [C]$

Proof (Truth-conditions of if A , C in w).

$$[if] = \lambda X \lambda Y n_w(X) \neq \emptyset \wedge e_{n_w(X)}(X) \subseteq [Y]$$

$$[if A] = \lambda Y n_w(A) \neq \emptyset \wedge e_{n_w(A)}(A) \subseteq [Y]$$

$$[if A, C] = n_w(A) \neq \emptyset \wedge e_{n_w(A)}(A) \subseteq [C] \quad \square$$

The first condition says that the inhibition of the antecedent must be successful, meaning it does not lead to an empty set of worlds. The second condition says that, by starting from this state of inhibition and by reconstructing the antecedent, all of the possible reconstructions are worlds in which the consequent is also true. In this semantics, the schema CS is no longer valid. Indeed, several reconstructions of the antecedent are considered. Hence, if the antecedent and the consequent are true in the initial world, this initial world is only one of the possibilities envisaged for the whole evaluation of the conditional. Other possibilities in which the antecedent is true but the consequent is false can also be obtained. This constraint is represented by a weaker semantic condition, compared to the one used in Chellas's system.⁵

An important consequence of this semantics is the non-monotonic behavior of the conditional. Indeed, the neutralization of two different sets of sentences can potentially lead to two different universes of projection. This feature is the main mechanism behind the invalidation of the strengthening of the antecedent: $A \rightarrow C \not\models (A \wedge B) \rightarrow C$. Consider the oft-used example "If Tweety is a bird, Tweety flies." For the premise, the conditional can be declared true because the most well-known type of bird is envisaged, meaning a flying bird. However, in the consequence "If Tweety is a bird and Tweety is a penguin, Tweety flies," the addition of a new antecedent forces us to reconsider a few assumptions, thus calling into question the hypothesis that we were talking about a flying bird. This mechanism is illustrated in Fig. 2 in which different sets of neutralized sentences lead to different expansion sets and ultimately to different truth-values.

Notice that this feature also explains that contraposition is invalid in this system: $A \rightarrow C \not\models \neg C \rightarrow \neg A$. Indeed, the set of neutralized sentences is $\{A\}$ for the premise and $\{\neg C\}$ for the consequence, implying that the two set relationships occur within two different universes of projection and are therefore not related. For that same reason,

⁵ If $w \in [A]$ and $n_w(A) \neq \emptyset$, then $w \in e_{n_w(A)}(A)$.

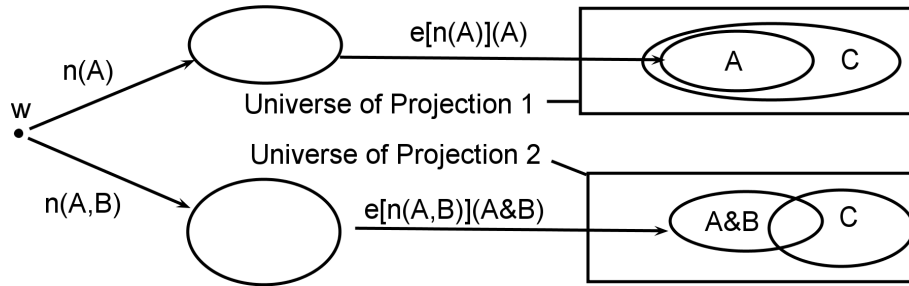


Fig. 2. Invalidity of the Strengthening of the Antecedent

transitivity is also invalid: $A \rightarrow B, B \rightarrow C \not\equiv A \rightarrow C$. Since the two premises respectively have $\{A\}$ and $\{B\}$ as inhibited sentences, their set-theoretic relationships again occur within two different universes of projection and cannot be combined to deduce the consequence. The invalidation of these three last schemas of reasoning is in line with the conditionals logics proposed by [30] and [19].

2 Then

I would now like to examine the semantics of *then*. It generally means *at that time*, *at that place*, or *in that case*, depending on the temporal, spatial, and logical interpretation. It embeds a phrase and relates it to a previous one. The preceding phrase can be either explicit such as the sentence that came just before or implicit. Hence, *then* is an anaphoric word that picks up the context of a prior element in order to place the subsequent element in this context. The referred element is made redundant in order to say that the subsequent element occurs during or just after these circumstances or this place. Consider the following example.

(5) We went to the beach. Then, we went to the museum.

The excursion to the sea is the first phase. *Then* indicates that the museum visit occurred just after. The succession of the two events is made explicit by the particle *then*. Without it, the temporal order between them would be left undefined, such as in “We went to the beach. We went to the museum.”

The temporal relationship is not necessarily one of succession, but sometimes one of cotemporality, such as in “I remember when I was young. Life was easier then.” In this case, the fact that life was easier is true within the stretch of time described as the narrator’s youth. The same ambiguity does not occur when spatial relationships are considered. The sentence “Standing beside my mother is my father, then my sister” suggests a relationship of succession, in which each person stands next to the other. But when we want to express a colocation between two elements, the particle *there* is preferred over the particle *then*, such as in the sentence “I remember this hill. A magnificent oak was standing there.”

Among the researchers who are interested in the semantics of *then*, some focus on the use of *then* inside conditional sentences and do not really consider the other cases

(see [16], [10] and [28]). Furthermore, they wait it to be combined with the conditional before giving a precise definition of its meaning. Their solution is therefore not sufficiently general to be satisfactory. I will examine their proposals in detail later on.

I shall now give a formal definition, remaining at the most general level of abstraction. As usual for the study of natural language, this definition is contextual. The precise elements on which *then* focuses depend on the situation. Furthermore, the precise result depends on the kind of reasoning, be it temporal, spatial, or logical.

Definition 2 (Meaning of Then). *The meaning of then is a function noted $g[then]_{C_X,Y}$ which takes two elements C_X and Y as inputs. C_X is the set of circumstances linked to the first element X . Y is simply the second element. The value of $g[then]_{C_X,Y}$ is $C_{X,Y}$. This means that Y is placed in the circumstances initially linked to X .*

Hence, the meaning of *then* is a function that incorporates the focused phrase into the context stated by the preceding discourse.

I would like to examine how this semantics applies to (5). The first sentence expresses that a group to which the narrator belongs went to the beach in the past. The circumstances here are temporal. The past circumstances can be noted by P and the sentence “We go to the beach” by X . Hence, the first input element is P_X . The second input element is Y , referring to the fact that the group went to the museum. The application of *then* results in the final relationship $P_{X,Y}$. More explicitly, Y occurs in the same past circumstances as X .

This definition involves the following consequence. The first input, of which the constituents are the circumstances, must precede *then*. Indeed, in order to be anaphoric, the referred element must have already occurred in the discourse. Concerning conditionals, the preceding clause is the antecedent. That explains why *then* can only be used when the protasis precedes the apodosis (6), and not in the opposite order (7), contrary to an *if A, C* conditional (8-9):

- (6) If it’s sunny, then I’m happy.
- (7) # Then I’m happy, if it’s sunny.
- (8) If it’s sunny, I’m happy.
- (9) I’m happy if it’s sunny.

This constraint does not generally hold for the second input. Sometimes, it can be asserted before *then*, for instance in “Weather was milder then.” But it can also appear afterward, such as in sentence (5). Another consequence of this definition is that the first input cannot be absent. Indeed, it is very difficult to randomly begin a discourse using the word *then*. Without an element to refer to, its presence would be deemed odd or cumbersome. I am not saying that it is totally impossible. One could choose to use it for rhetorical reasons. From a logical point of view, however, this missing element would render the usual function of *then* inapplicable. Similarly, the second input is generally present. If a speaker ends a sentence without providing this element, it forces the listener to guess it. For instance, it can be used to scare a disobedient child, such as in “Be careful! If you do that, THEN ...” The consequences are such a frightening prospect that they cannot be uttered.

Then is primarily used temporally and logically. The acquisition of these two different forms of reasoning must in some way be linked. [18] sums up the studies of the

developmental acquisition of conditionals in the following way. Young children interpret them as conjunctions, older children as biconditionals, and adults as conditionals. Notice that the meaning of a logical conjunction does not include a relationship of succession because $A \wedge B$ is equivalent with $B \wedge A$. The biconditional is also a relationship of equivalence between $A \equiv B$ and $B \equiv A$. This symmetry does not hold for the conditional, and this part of its meaning is probably acquired through the relationship of succession in temporal reasoning. Jean Piaget's four stages of cognitive development confirm this relationship between both types of reasoning (see [23]). According to Piaget, during the *concrete operational stage*, which occurs from ages 7 to 11, the child learns to use several operations and classify elements among them. At the end of this stage, the child acquires the spatio-temporal system of common-sense reasoning. During the final stage, known as the *formal operational stage*, which occurs starting at the age of 12, the adolescent is able to reason abstractly and, in particular, to perform hypothetical reasoning. Hence, the temporal meaning of *then* is probably learned before it is applied to conditional reasoning.

This might explain why several researchers in linguistics, such as [27], [15] and [33] favor the temporal meaning over other uses of the word. Their main discovery is that the signification of *then* depends on its position. In a clause-initial position, *then* entails a reading of succession. In a clause-final position, it is understood as having a cotemporal meaning. Thompson tries to explain this dichotomy by using the difference between the event time and the reference time used by [26] to describe the temporal discourse. She argues that, in a final position, *then* focuses on the Verb Phrase and links its event time with the event time of the preceding clause. Both event times are therefore cotemporal. On the contrary, when *then* is in an initial position, it focuses on the Inflectional Projection and links its reference time to the reference time of the preceding clause. In this case, both event times are not linked and are therefore not cotemporal, but ordered. The weakness of this argument lies in this last inference. As soon as two event times are not linked by *then*, there is no reason why any temporal ordering should occur between them and why this ordering should be a relationship of succession. The exact mechanism allowing the cotemporal or the ordered reading, depending on the position of the adverb, is at best described although not explained in this setting.

The difference in the meaning of the temporal *then* can be more easily explained in the following way. The ordering of the words in the discourse is similar to the temporal ordering of the events referred by the words. With *then* in an initial position comes the following form: A then B. Because *then* is an anaphoric word of the preceding clause, the ordering of these elements is therefore: A A B. In this case, B clearly occurs after A and succeeds to it. On the contrary, when *then* is in the final position, the pattern is as follows: A, B then. Again, as *then* has an anaphoric function, the exact ordering of the elements is: A B A. The element B is flanked by two elements A, both of which are therefore cotemporal. This iconic explanation is based on the similarity between the ordering in the discourse and the temporality of events and is simpler than Thompson's attempt.

3 If Then

We arrive now to the main objective of this paper: obtaining a compositional meaning for *if then* conditionals, based on the definitions given for the meaning of *if* and *then* in the two preceding sections. Considering the combination of *if* and *then*, it becomes clear that the association between the particle *if* and the antecedent constitutes the base for the set of circumstances that is the first input of the semantics of the particle *then*. It is also clear that the consequent is the second input of this function. However, this compositional semantics leads to the following question: what exactly are these circumstances that constitute the first input? I argue that, in this case, the set of circumstances is constituted by the possibilities representing the inhibition of the antecedent, which occurs during the first phase of evaluation of a conditional. For the record, this inhibition is also called neutralization and is represented by the function n in the formal semantics. The first reason for this choice is that, just before the assertion of the *then*, only the antecedent has been uttered. This means that only the inhibition phase can be completed at this time. Indeed, the reconstruction phase is always missing the consequent to be proceeded. The second reason for this adoption is that the neutralization can be considered as the context determining the universe of projection in which the final evaluation will be conducted. Indeed, in this semantics, a different set of inhibited sentences leads to a different universe of projection. Being the context of a conditional, the set of possibilities envisaged at the end of the inhibition phase is therefore the first input of the meaning function of the *then*.

Definition 3 (First Input of Then when applied to If). *For a conditional, the first input of the meaning function of then is the set $n_w(A)$ with n being the neutrality function, w the starting world of the evaluation, and A the antecedent of the conditional.*

With this definition, placing the second element within the context of the first element means that the consequent is also among the set of inhibited sentences. It is added to the antecedent to form the set of sentences that are explicitly neutralized. Thus, the conditional *if A then C* receives the following meaning.

Theorem 2 (Truth-conditions of if then).

$\models_w \text{if } A \text{ then } C$ iff

- i) $n_w(A, C) \neq \emptyset$
- ii) $e_{n_w(A, C)}(A) \subseteq [C]$

Proof (Truth-conditions of if A then C in w).

$$[\text{if}] = \lambda X \lambda Y n_w(X) \neq \emptyset \wedge e_{n_w(X)}(X) \subseteq [Y]$$

$$[\text{if } A] = \lambda Y n_w(A) \neq \emptyset \wedge e_{n_w(A)}(A) \subseteq [Y]$$

$$[\text{if } A \text{ then}] = \lambda Y n_w(A, Y) \neq \emptyset \wedge e_{n_w(A, Y)}(A) \subseteq [Y]$$

$$[\text{if } A \text{ then } C] = n_w(A, C) \neq \emptyset \wedge e_{n_w(A, C)}(A) \subseteq [C] \quad \square$$

The addition of *then* modifies the basic semantics by incorporating the consequent into the set of inhibited sentences. In the simple *if A, C* form, this neutralization was optional. It now becomes mandatory with the presence of *then*. This means that if the consequent reappears in each world in which the antecedent is true, both of them are

linked in a stronger way. Indeed, in the *if A, C* form, the consequent was not necessarily inhibited. Its presence in the A-worlds can therefore be explained by a potential non-inhibition, which has nothing to do with the presence of the antecedent. With the *if A then C* form, the consequent is no longer believed to be true or false at the end of the inhibition phase. If the conditional is true, the reappearance of the consequent in each A-world means that its occurrence is in one way or another linked to the occurrence of the antecedent. Their copresence can no longer be fortuitous.

I would like now to review the differences between this proposal and the most well-known approaches to the meaning of *if then* conditionals. The only really formal predecessor to this theory is the one proposed by [2], who argues for a compositional analysis, but one that is restricted to counterfactuals. Indeed, Alonso-Ovalle surprisingly chooses the consequent and more precisely its inner *would* item as bearing the main meaning of conditionality. However, an extension of this analysis to the indicative case seems doomed to fail because this mood does not contain such an auxiliary modal verb. Furthermore, this choice does not dovetail with the general strategy present in most natural languages (particularly in English), which is to modify the antecedent and not the consequent by using the word *if* in order to signal a conditional sentence ([5]).

The biconditional approach is defended by [8], who argue that *then* locates the obtainment of the consequent in the event associated with the antecedent. By contrast, this obtainment does not hold in other alternatives. As a consequence (see p.145), “*then* allows for the ‘ $\sim P, \sim Q$ ’ inference precisely because it is a deictic referent for a mental space. It consequently both brings up the idea that P is the unique mental space in which Q is ‘located’ and explicitly marks the relevant connection between P and Q as causal and/or sequential.” Dancygier and Sweetser adopt both the “P, Q” and the “ $\sim P, \sim Q$ ” inferences, thus defending a biconditional interpretation of the *if then* conditional. This position has some problems. First, when the biconditional is explicitly asserted in natural language, it is with a different form: *if and only if*. There is no reason why these two forms, which differ syntactically, are equivalent from a semantic point of view. This equivalence would at least need to be justified. Another problem with a biconditional interpretation is that the occurrence of the consequent entails the occurrence of the antecedent, as in the following inference: *A iff C, C. Thus A*. But this inference is not always valid for the *if then* form. For instance, speaking about tennis, I could say, “This is a match point. If she wins the next point, then she wins the match.” But subsequently knowing that she won the match, I cannot conclude that she won this match point precisely because she perhaps needed some others. For both of these reasons, the biconditional interpretation is implausible.

The last important approach that I would like to examine is defended by [16], [10] and [28], with some variations. For them, the *if then* conditional carries the following implicature: there are some cases in which the negation of the antecedent is present with the negation of the consequent. As Iatridou says, “Then ‘carries the presupposition’ that ‘ $\sim p$ is compatible with $\sim q$ ’, which is weaker than ‘ $\sim p$ implies $\sim q$ ’.” This approach has the advantage of not implying a biconditional interpretation. In Schlenker’s version, *then* is a strong pronoun, and “in the case of then in *if p, then q*, the implicature is that some non-*p* worlds are non-*q* worlds.” This is Iatridou’s idea expressed in a possible world semantics.

While this idea works in a lot of cases, there are counterexamples. For instance, I can say, “All these sweets are green. If they are green, then they are peppermint candies.” In that case, the speaker knows perfectly well that there are no non-green candies in the bag. The implicature is therefore implausible. However, the main problem of this approach is its direct usage of the negation in the implicature. When *then* is used outside conditionals, the negation is not a part of its meaning. Its import for the implicature is therefore completely artificial. If we again take our example (5) “We went to the beach. Then, we went to the museum,” we see that the temporal interpretation only carries the idea of succession. If we also consider *then* in isolation, it is simply an anaphoric word, the meaning of which does not contain the idea of negation. To sum it up, the issue faced by this approach is that the use of negation for conditional cases cannot be derived from a more general theory of *then*.

On the contrary, the theory I am proposing in this paper makes it possible to formally derive a particular property for *if then* conditionals, without referring to any additional implicature or other pragmatic principle. As Iatridou and Schlenker argued, along with Dancygier and Sweetser, this effect concerns the relationship entertained by the negation of the antecedent and the negation of the consequent, but this relationship is not exactly what they proposed. Indeed, the new schema of reasoning that is introduced in this semantics is contraposition: from “If A, then C”, we can immediately deduce “If not C, then not A”.

- (10) If I have money, then I buy bread.
 (11) If do not buy bread, then I have no money.

Hence, what is predicted is that all the cases in which the consequent is false are cases in which the antecedent is false. There is therefore no biconditional interpretation. Furthermore, there is no obligation for such cases to exist. Intuitively, the validity of contraposition is obtained in the following way. The truth-conditions obtained for “if A then C” implies that the inhibition of A and C was successful. Inhibiting sentence A leads to no longer believing it to be true or false. Hence, the inhibition of a sentence is equivalent to the inhibition of its negation. Therefore, the inhibition of A and C is equivalent to the inhibition of $\neg C$ and $\neg A$. They define the same context of evaluation. Inside this universe of projection, all the A-worlds are C-worlds. As all these worlds are bivalent concerning the antecedent and the consequent, all the $\neg C$ -worlds are $\neg A$ -worlds. This therefore makes it possible to conclude that “if $\neg C$ then $\neg A$ ” is true.⁶

4 A Defense of Contraposition for If Then Conditionals

The idea that contraposition is a valid schema of reasoning for *if then* conditionals goes against what is usually argued in the related literature, which is why I shall now present a detailed defense of this position. Some authors, such as [35], [36], [13], [14] and [32], reject the usual counterexamples to contraposition by using pragmatic principles.

⁶ More formally, in the settings defined in Appendix A by [38], from if A then C, we have $n_w(A, C) \neq \emptyset$ and $e_{n_w(A, C)}(A) \subseteq [C]$. From $n_w(A, C) \neq \emptyset$ and (neut), we obtain $n_w(A, C) = n_w(\neg C, \neg A)$ and $n_w(\neg C, \neg A) \neq \emptyset$. By $e_{n_w(A, C)}(A) \subseteq [C]$ and set-theoretic equivalence in the universe of projection U, we obtain $e_{n_w(\neg C, \neg A)}(\neg C) \subseteq [\neg A]$.

They argue that these counterexamples are conversationally infelicitous and cannot be counted against their theories. They also apply the same mechanism to reject counterexamples to other patterns of inferences such as the strengthening of the antecedent or transitivity. I adopt a different position by considering the counterexamples to these different schemas of reasoning to be generally correct and that they should not be removed by a call to pragmatic principles. In particular, the invalidity of the strengthening of the antecedent shows that the conditional is a non-monotonic connective, and this invalidity should be kept in a correct theory of hypothetical reasoning. Furthermore, there is nothing to say against most of the counterexamples to contraposition, except that they do not apply to the *if then* form.

I would like to illustrate this point using a concrete case. [4] gives the following definition of a conditional early on in his study (see p. 3): “An item is a conditional if it is expressed by an English sentence consisting of ‘If’ followed by an English sentence followed by ‘then’ followed by an English sentence.” However, when it later comes time to give a counterexample to contraposition (see p. 172), he makes the following curious choice: “(Even) if the British and Israelis had not attacked the Suez Canal in 1956, the Soviets would (still) have invaded Hungary later in the year.” Hence, Bennett considers conditionals to be part of the *if then* form but uses a concessive conditional (an *even if* form) to provide a counterexample to contraposition. This surprising choice can be explained by considering that most theories of conditionals do not provide a different semantics when the *if* is enriched by additional particles such as *even* and *then*. However, as soon as an analysis takes into account these additional particles, the assimilation between the *even if* and the *if then* forms cannot be maintained. Furthermore, if the validity of contraposition differs for these two forms in natural language, a formal theory not taking this level of analysis into account misses an essential point. It is therefore primordial to check whether the invalidity of contraposition for *if then* conditionals is a reality in everyday reasoning. To do so, I would like to give an overview of the related literature to see whether a convincing counterexample to this schema can be found.

4.1 Cases Not Using the *If Then* Form

Almost all the counterexamples to contraposition found in books or articles on the subject do not concern *if then* conditionals. As seen in [4], the majority of them concern concessive conditionals. Indeed, they can be reformulated along the “even if” pattern without their meaning being modified, as shown below by adding parentheses around the word *even* when needed.⁷

- (12) (Even) if Boris had gone to the party, Olga would still have gone.
- (13) (Even) if it rains tomorrow there will not be a terrific cloudburst.
- (14) My car would still be white even if the maple tree in my front yard died.
- (15) (Even) if Goethe hadn’t died in 1832, he would still be dead now.
- (16) (Even) if you open the refrigerator, it will not explode.

Notice that these sentences are taken from some of the most famous works in the field. Furthermore, some of their authors - for instance Lewis or Adams - were primarily

⁷ (10) to (14) are respectively from [19], [1], [24], Kratzer cited in [11] and [20].

uninterested in *even if* conditionals, and their usage precisely at this point is rather suspicious. If these authors did not find any sufficiently convincing counterexample to the *if then* form and were forced to use an *even if* form, it is surely because such a sentence is not so easily found.

A variant of this dismissal of the contraposition uses relevant conditionals (also referred to as “biscuit” conditional or “nonconditional” conditional). The following are some examples of this type sentences, taken from [3], [12] and [17] respectively.

- (17) If you you are hungry, there are biscuits on the table.
- (18) If you don’t mind, I’m trying to read.
- (19) If you had needed some money, there was some in the bank.

Again, these sentences cannot constitute counterexamples to the *if then* form because the particle *then* cannot be added to them felicitously. Furthermore, it is widely agreed that relevance conditionals deserve their own particular treatment.

4.2 Simple Cases of the *If Then* Form

To my knowledge, there are only two counterexamples to contraposition that adopt the *if then* form in the literature.⁸ The first one, given by [30], is well known because it was published in the first article that presented a possible world semantics for conditionals invalidating contraposition. However, this example has not been reused in later works by other authors, showing that it is certainly deficient. The sentence is as follows:

- (20) If the U.S. halts the bombing, then North Vietnam will not agree to negotiate.

The sentence sounds odd and is difficult to understand. It can be explained by recalling what happened during the Vietnam War. The speaker does not mean that as long as the U.S. continues the bombing, North Vietnam will agree to negotiate and that this will change if the U.S. takes the opposite stand. Negotiations between the U.S. and North Vietnam took place in 1973, after ten years of intense bombing and only once the U.S. was considered to have lost the war, as far as public opinion was concerned. Here, the speaker wants to offer another meaning: bringing a halt to the bombing is a favorable factor for sparking negotiation, but is on its own insufficient for bringing North Vietnam to the negotiating table. A felicitous expression of this relationship would be “*Even if* the U.S. halts the bombing, North Vietnam will not agree to negotiate.”

Stalnaker thus makes a grammatical error using an *if then* form when the correct formulation requires an *even if* form. Some could consider it completely licit to use the *if then* form to carry the meaning of a concessive conditional. Indeed, language is governed not by explicit rules, but by practice. As soon as the audience is able to decipher the intended meaning, the exact syntactic form used is unimportant. In this case, this argument does not hold. Indeed, the psychological experiment presented by [9] shows that, when an antecedent is deemed as not presenting evidence for the consequent (the concessive meaning), almost all subjects do not accept the *if then* form as a correct assertion. Both expressions therefore differ in meaning and cannot be freely exchanged in

⁸ There are only two counterexamples in the major works. The literature on conditionals is so huge that nobody can claim to have read all the articles or books on this subject.

a conversation without running the risk that most listeners will not understand the exact meaning or will simply reject the utterance. Stalnaker, surely knowing his counterexample to be deficient, adopts another one in a later book on conditionals. In his [31] study, the example becomes “If my dog were a purebred, his father would be a mutt,” but as Stalnaker himself acknowledges, “One could reject the counterexample on the grounds that the conditional contraposed is an ‘even if’ conditional.” I completely agree with Stalnaker here.

[25] presents another counterexample to contraposition, although without defining any clear context.

(21) If we take the car then it won’t break down *en route*.

The first problem with this pseudo-counterexample is that it is easy to imagine a context in which the contraposition is valid. Imagine that two groups can take the car to drive on a very difficult trail and that only the group to whom the speaker belongs has a sufficiently good driver to do this safely. Learning that there was an accident, the following deduction is possible: “If the car broke down *en route*, then you didn’t take it. The driver was a member of the other group.” Priest perhaps had another context in mind, in which there is only one group of people. But now, if they do not take the car, it is certain that it will not also break down *en route* because the vehicle safely remains parked in the garage. The conditional relationship that the speaker wants to express is now a concessive one that would be better expressed using an *even if*: “Even if we take the car, it won’t break down *en route*”. The same problem as in Stalnaker’s case therefor arises. The use of the *if then* form is deficient because the speaker wants to carry a concessive meaning. The counterexample therefore cannot be retained.

4.3 Cases with Modals

The last possibility for invalidating contraposition for *if then* conditionals is to incorporate modals. The following sentence is a slight modification of an example against the modus tollens initially presented in [39].

(22) If there is a break-in, then the alarm always sounds.

From the hypothesis that the alarm does not always sound, we cannot conclude that there is no break-in. Hence, the contraposition is invalid in this case. However, this invalidation is incorrect because it is based on an ambiguity. Since the consequent is the main clause of the whole sentence, the association between an adverb and its verb can result in two different scopes of application. The word “always” can be considered as either applying only to the consequent (“If there is a break-in, then always the alarm sounds”) or to the whole conditional (“Always, if there is a break-in, then the alarm sounds”). In most cases, the correct interpretation is to allow the adverb to apply to the whole sentence. For sentence (22) and with a large scope of application for the adverb, the contraposition now becomes “Always, if the alarm does not sound, then there is no break-in,” which is perfectly correct. Yalcin’s counterexample is therefore deficient because it is based on a narrow scope of interpretation for the adverb, for which a large scope of interpretation is necessary. This issue is well known in modal logic referred to

as the (conditional) Modal Fallacy by [6]. It is usually presented with explicitly modal adverb, such as in the following reformulation of (22):

(22') If there is a break-in, then the alarm necessary sounds.

The correct formal translation of (22') is $\Box(A \rightarrow C)$ and not $A \rightarrow \Box C$, with \Box the symbol for necessity and \rightarrow the symbol for the *if then* conditional.

4.4 Advantages

From this examination of earlier attempts, I have shown that, in order to be successful, a counterexample to contraposition for *if then* conditionals should involve two things: components that are sufficiently simple to not embed modals and an antecedent and a consequent of which the relationship is not concessive or one of relevance. I cannot prove that such a counterexample is impossible because new sentences are invented everyday in natural language. But the fact that, after fifty years of investigation in conditional logic, no convincing *if then* conditional sentence has been found to invalidate this reasoning leads me to think that this is a hopeless task. However, even if I am wrong on this point, the present semantics has its own merits. This theory is the first possible worlds semantics that manages to separate the validity of contraposition and strengthening of the antecedent. Contraposition is therefore not a non-monotonic reasoning, as argued by [11] for instance. Indeed, a non-monotonic inference involves a modification of context because a new sentence changing the initial situation is introduced. For instance, in the strengthening of the antecedent $A \rightarrow C \neq (A \wedge B) \rightarrow C$, the new hypothesis B makes the interpretation of the hypothesis A different between the premise and the consequence. In the transitivity $B \rightarrow C, A \rightarrow B \neq A \rightarrow C$, the second premise contains A, which is new and changes the context in which the sentence B must be interpreted compared to the first premise. Hence, the link between the two premises is impossible. There is no such introduction of a new sentence in contraposition. Between the premise and the conclusion, the only new element is the introduction of the negation and this cannot produce a new context. On the contrary, this only entails the consideration of the negatives cases in the initial context.

[31] argues that the validity of contraposition always entails the reintroduction of the strengthening of the antecedent. The proof is as follows:

- (23) $A \rightarrow C$ (hypothesis).
- (24) $\neg C \rightarrow \neg A$ (From 23 by contraposition)
- (25) $\neg C \rightarrow (\neg A \vee \neg B)$ (From 24 by expansion of the consequent)
- (26) $(A \wedge B) \rightarrow C$ (From 25 by contraposition)

This proof needs an additional principle: the expansion of the consequent. However, this last inference is not valid in our system: $A \rightarrow B \neq A \rightarrow (B \vee C)$. Since the conditional in the consequence contains more sentences than the conditional in the premise, the consequence needs the inhibition of a larger set of sentences, and its final evaluation is done in another context compared to the premise. My semantics thus introduces a principle of relevance between the antecedent and the consequent. Each part of the consequent must be related in some way to at least one part of the antecedent to make the

conditional true. This choice makes it possible to explain why *if then* is the privileged form used to express causal reasoning and why the following sentence sounds false in this context:

(27) If I'm hungry, then I eat a banana or Asia is the biggest continent.

My hunger and the size of Asia are not causally linked. As a consequence, most people would consider sentence (27) to be wrong or odd for describing a causal relationship. However, notice that with the simple *if A, C* form, the reasoning is correct. The relationship being more tenuous, the schema $A \rightarrow B \vDash A \rightarrow (B \vee C)$ is valid.

The last advantage of the admission of contraposition is that it is now possible to understand why the particle *then* cannot be added in some conditionals. Indeed, when the contraposition fails, the addition of *then* is infelicitous. This is the case for relevance (biscuit) conditionals. The same rule also holds for conditionals in which the antecedent exhausts the universe, such as in [16]'s proposal "If John is dead or alive, Bill will find him." In this case, the negation of the antecedent cannot be envisaged because it corresponds to an empty set of worlds, and there is no reason to utter contraposition. Finally, contraposition is not valid for concessive conditionals which explains the initial enigma of this paper: how come an *even if* conditional cannot contain the particle *then*.

References

- [1] Adams, E.W.: The Logic of Conditionals. D. Reidel Publishing Co., Dordrecht (1975)
- [2] Alonso-Ovalle, L.: Alternatives in the Disjunctive Antecedents Problem. In: Chang, C.B., Haynie, H.J. (eds.) Proceedings of the 26th West Coast Conference on Formal Linguistics. Somerville, MA (2008), Cascadia Proceedings Project
- [3] Austin, J.L.: Ifs and Cans. In: Urmson, J.O., Warnock, G.J. (eds.) Philosophical Papers, pp. 153–180. Oxford University Press, Oxford (1961)
- [4] Bennett, J.: A Philosophical Guide to Conditionals. Clarendon Press - Oxford (2003)
- [5] Bhatt, R., Pancheva, R.: Conditionals. In: Everaert, M., Van Riemsdijk, H. (eds.) The Blackwell Companion to Syntax, pp. 638–687. Wiley (2006)
- [6] Bradley, R., Swartz, N.: Possible Worlds: An Introduction to Logic and Its Philosophy. B. Blackwell (1979)
- [7] Chellas, B.: Basic Conditional Logic. Journal of Philosophical Logic 4(2), 133–154 (1975)
- [8] Dancygier, B., Sweetser, E.: Mental Spaces in Grammar : Conditional Constructions. Cambridge University Press, Cambridge (2005)
- [9] Douven, I., Verbrugge, S.: Indicatives, Concessives, and Evidential Support. Thinking & Reasoning 18(4), 480–499 (2012)
- [10] von Fintel, K.: Restrictions on Quantifier Domains. Ph.D. thesis, University of Massachusetts, Amherst, MA (1994)
- [11] von Fintel, K.: Counterfactuals in a Dynamic Context. In: Kenstowicz, M. (ed.) Ken Hale: a Life in Language, pp. 123–152. The MIT Press, Cambridge, Massachusetts (2001)
- [12] Geis, M.L., Lycan, W.G.: Nonconditional Conditionals. Philosophical Topics 21(2), 35–56 (1993)
- [13] Gillies, A.S.: Epistemic Conditionals and Conditional Epistemics. Noûs 38(4), 585–616 (2004)
- [14] Gillies, A.S.: On Truth-Conditions for If (but not quite Only If). Philosophical Review 118(3), 325–349 (2009)

- [15] Glasbey, S.R.: Distinguishing between Events and Times: Some Evidence from the Semantics of Then. *Natural Language Semantics* 1(3), 285–312 (1993)
- [16] Iatridou, S.: On the Contribution of Conditional 'Then'. *Natural Language Semantics* 2(3), 171–199 (1994)
- [17] Johnson-Laird, P.N.: Conditionals and Mental Models. In: Traugott et al. [34], pp. 55–76
- [18] Johnson-Laird, P.N., Byrne, R.M.: Conditionals: a Theory of Meaning, Pragmatics, and Inference. *Psychological Review* 109(4), 646–678 (2002)
- [19] Lewis, D.K.: *Counterfactuals*. Harvard University Press, Cambridge, Massachusetts (1973)
- [20] Lycan, W.G.: *Real Conditionals*. Oxford University Press, Oxford (2001)
- [21] Matalon, B.: Etude Génétique de l'Implication. In: Piaget, J. (ed.) *Etudes d'Epistémologie Génétique XVI*. PUF, Paris (1962)
- [22] Nute, D.: *Topics in Conditional Logic*. Reidel, Dordrecht (1980)
- [23] Piaget, J., Gruber, H., Vonèche, J.: *The Essential Piaget*. J. Aronson (1977)
- [24] Pollock, J.L.: *Subjunctive Reasoning*. D. Reidel Publishing Co., Dordrecht (1976)
- [25] Priest, G.: *An Introduction to Non-classical Logic: From If to Is*. Cambridge University Press (2008)
- [26] Reichenbach, H.: *Elements of Symbolic Logic*. Republished 1980, Dover Publications (1947)
- [27] Schiffrin, D.: Anaphoric Then: Aspectual, Textual, and Epistemic Meaning. *Linguistics* 20, 753–792 (1992)
- [28] Schlenker, P.: Conditionals as Definite Descriptions (a Referential Analysis). *Research on Language and Computation* 2(3), 417–462 (2004)
- [29] Skovgaard-Olsen, N., Singmann, H., Klauer, K.C.: The Relevance Effect and conditionals. *Cognition* 150, 26–36 (2016)
- [30] Stalnaker, R.C.: A Theory of Conditionals. In: Rescher, N. (ed.) *Studies in Logical Theory*, pp. 98–112. Basil Blackwell Publishers, Oxford (1968)
- [31] Stalnaker, R.C.: *Inquiry*. The MIT Press, Cambridge, Massachusetts (1984)
- [32] Starr, W.: A Uniform Theory of Conditionals. *Journal of Philosophical Logic* 43(6), 1019–1064 (2014)
- [33] Thompson, E.: The Temporal Structure of Discourse: the Syntax and Semantics of Temporal Then. *Natural Language and Linguistic Theory* 17, 123–160 (1998)
- [34] Traugott, E., ter Meulen, A., Reilly, J., Ferguson, C. (eds.): *On Conditionals*. Cambridge University Press, Cambridge, England (1986)
- [35] Veltman, F.: *Logics for Conditionals*. Ph.D. dissertation, University of Amsterdam, Amsterdam (1985)
- [36] Veltman, F.: Data Semantics and the Pragmatics of Indicative Conditionals. In: Traugott et al. [34], pp. 147–167
- [37] Vidal, M.: *Conditionnels et Connexions*. Ph.D. dissertation, Institut Jean Nicod, E.H.E.S.S., Paris (2012)
- [38] Vidal, M.: A Compositional Semantics for 'Even If' Conditionals. *Logic and Logical Philosophy* (in press)
- [39] Yalcin, S.: A Counterexample to Modus Tollens. *Journal of Philosophical Logic* 41, 1001–1024 (2012)