Abstract

This paper examines how moods are used in declarative conditionals. Contrary to what is usually defended in the literature, the divide between indicative and subjunctive is unable to account for all the possible cases present in natural language. Therefore, the analysis goes down to the level of the antecedent and the consequent in order to examine their local and global interactions with moods. A formal semantics split into two layers is presented in order to model all the possible cases. The lower layer is the semantics of the conditional and the upper layer is the semantics for the indicative, subjunctive and conditional moods. Finally, it is shown that this proposal provides a unified solution to three puzzles: the Oswald-Kennedy example, the direct argument and the Gibbardian stand-offs.

Keywords: conditional logic, if, indicative, subjunctive, moods

1 Introduction

How do conditionals interact with moods? The classical line of division separates the conditionals into two groups: the indicative group and the subjunctive group. Indicative conditionals are hypothetical sentences in which both components are at the indicative mood. Subjunctive conditionals can receive two different forms. In the first form, the antecedent is a past subjunctive, a tense which only differs from the past indicative in modern English by replacing ‘was’ by ‘were’. In the second form, the antecedent is conjugated in the past perfect (had + past participle), which is an indicative mood. However, this use is assimilated to a subjunctive form for three reasons: the antecedent can be reformulated with the locution ‘were to have + past participle’, it allows an inversion and it is often translated into a mood which is explicitly subjunctive in other languages. For both forms of subjunctive conditional, the consequent is expressed through the conditional mood by usually using the auxiliary would.

(1) INDICATIVE: If you need some money, you can use an ATM.

(2) SUBJUNCTIVE 1: If you were in need of money, you would use an ATM.

(3) SUBJUNCTIVE 2: If you had needed some money, you would have used an ATM.
(4) **SUBJUNCTIVE 2 reformulated:** If you were to have needed some money, you would have used an ATM.

In the field, this distinction between indicative and subjunctive is considered important for different reasons. First, counterfactuals are usually expressed with the subjunctive but Chisholm (1946) and Goodman (1947) showed that the material conditional from classical logic cannot correctly model it. By contrast, several pragmatic defenses of the modeling of the indicative by the material conditional exist, for instance following Grice (1967). Second, both forms sometimes differ concerning their truth-values, as shown by the famous example issued from (Adams, 1970):

(5) If Oswald did not kill Kennedy, then someone else did.

(6) If Oswald had not killed Kennedy, then someone else would have.

Finally, the technical treatments of indicative conditionals generally favor probabilities whereas subjunctive conditionals are often modeled by using possible worlds. All these differences lead some authors to consider that both types must receive a separate semantic treatment (Lewis, 1973; Adams, 1975; Gibbard, 1981; Edgington, 1995; Bennett, 2003). On the contrary, other researchers (Stalnaker, 1968; Lycan, 2001; Starr, 2014; Khoo, 2015) defend a unified theory of conditionals, partly because the word *if* is such a basic element of language that it must keep a common meaning in these two contexts of usage.

However, this division presents a major difficulty. These two groups cannot represent all possible conditionals. Indeed, a conditional with an antecedent in the subjunctive mood and a consequent in the indicative mood can also be asserted, as shown by (Johnson-Laird, 1986):

(7) **SUBJUNCTIVE/INDICATIVE:** If you had needed some money, there was some in the bank.

Thus, the classical division between indicative and subjunctive is insufficient to take into account all the combinations of moods with conditionals. Furthermore, no previous approach clarifies which are the combinations allowed in natural language. But this is a minimal requirement for any complete theory of the interactions between conditionals and moods! In this paper, I will tackle this issue for declarative sentences, not tacking into account questions and orders. As a consequence, the imperative mood will not be considered. I will show that conditional sentences can be modeled through a unique basic semantics, supplemented by an additional layer taking into account the moods. The interaction between these two layers makes it possible to explain why only some combinations are allowed in natural language. Furthermore, this semantic approach will also describe how the meaning of the same conditional can partly differ by simply changing the moods of its components.

In section 2, I will empirically examine what are the possible combinations of moods and conditionals allowed in English. In section 3 and section 4, I will detail how the semantics of *if* interacts with the semantic of the moods, respectively for normal and relevant (biscuit) conditionals. Finally, in section 5, I will show that this approach provides a unified solution to three famous puzzles in the field: the Oswald-Kennedy example, the direct argument and the Gibbardian stand-offs.

## 2 Possible and Impossible Combinations

In this section, I will examine what are the possible combinations of moods for declarative conditionals in English. The data of this inquiry are mainly gathered by cross-checking the different examples from literature, personal thought experiments and internet searches. Only three moods can be used in declarative conditionals: the indicative, subjunctive and conditional moods. The indicative mood can be found in the antecedent and in the consequent, the subjunctive is only present in the antecedent, and the conditional mood is only used in the consequent. No other moods are represented in declarative hypothetical sentences. This first review leaves us with four possible combinations to examine.
I will consider the indicative mood as the default usage and do not use any special symbol to mark it. On the other hand, I will use the symbol ‘◦’ to signal both the subjunctive mood in the antecedent and the conditional mood in the consequent. Indeed, these two moods have a similar function: both denote that the sentence goes against what is commonly assumed in the context of conversation. Furthermore, many authors qualify the consequent with a conditional mood as a subjunctive consequent. Hence, we have the following four possible combinations of moods in a conditional sentence:

(a) If A, C (indicative/indicative)
(b) If ◦A, C (subjunctive/indicative)
(c) If A, ◦C (indicative/conditional)
(d) If ◦A, ◦C (subjunctive/conditional)

However, a careful examination of the previous sentence (7) “If you had needed some money, there was some in the bank” shows that it is a relevant conditional, also called biscuit conditional from the famous example found by Austin (1961). Indeed, it is not the truth of the consequent but its relevance for the context of assertion which depends on the truth of the antecedent. By contrast, the same combination of mood (subjunctive in the antecedent and indicative in the consequent) cannot be found for normal (non-relevant) conditionals. Hence, we have to examine our four possibilities two times: for the normal conditionals and for the relevant conditionals.

Admittedly, the combination (a) allows both types of interpretations. Here are two examples, the first for normal and the second for relevant conditionals:

(8) If you take the wrong direction, you will be late.

(9) If you are thirsty, there is some fresh water in the fridge.

The combination (b) is not possible for normal conditional, as shown by the following first sentence, but is possible for a relevant interpretation, as shown by the second conditional:

(10) # If it had rained, I stayed at home.

(11) If you were thirsty, there was some fresh water in the fridge.

The combination (c) is not possible for normal conditionals1, in contrast with the combination (d) which is the most common way to express counterfactuals:

(12) # If you take the wrong direction, you would be late.

(13) If you had needed some money, you would have taken your wallet.

The case is less clear for the application of the combinations (c) and (d) to relevant conditionals. Let us take our basic example of such sentence “if you are hungry, there are biscuits on the table.” By giving it the forms (c) or (d), the sentence now carries a normal interpretation. The realization of the consequent depends on the realization of the consequent:

(14) If you are hungry, there would be biscuits on the table.

(15) If you had been hungry, there would have been biscuits on the table.

1 Few examples of such mixture can be found on internet, for instance “if it wasn’t for your arrogance, you would have gotten your promotion.” But these occurrences exhibit specific locutions such as “if it wasn’t for” that are not grammatically correct in this context. An accurate reformulation of this sentence would be “if it were not for your arrogance, you would have gotten your promotion” or “if you were not so arrogant, you would have gotten your promotion.”
Swanson (2013) notices that many authors consider relevant conditionals to be permissible only with an indicative form. However, he also shows with the following examples that this conventional wisdom is mistaken. Sometimes, relevant conditionals can adopt the forms (c) and (d):

(16) I want to vacation in a posh hotel in London. We would have tea every afternoon, and there would be biscuits on the sideboard, if you’re into that sort of thing.

(17) I want to vacation in a posh hotel in London. We would have tea every afternoon, and there would be biscuits on the sideboard, if one were so inclined.

Hence, we have two different phenomena at work. Taken in isolation, a relevant conditional cannot be reformulated with a conditional mood in the consequent, without taking a normal interpretation. In contrary, by adding a detailed scenario, this transformation is possible. We will have to explain this difference of behavior.

Table 1 sums up the height possibilities with their respective status. We mark the two last cases concerning the relevant conditional ‘OK/KO’ to signal that this construction is sometimes possible and sometimes impossible.

<table>
<thead>
<tr>
<th></th>
<th>If A, C</th>
<th>If ◦A, C</th>
<th>If A, ◦C</th>
<th>If ◦A, ◦C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal Conditional</td>
<td>Case 1: OK</td>
<td>Case 2: KO</td>
<td>Case 3: KO</td>
<td>Case 4: OK</td>
</tr>
<tr>
<td>Relevant Conditional</td>
<td>Case 5: OK</td>
<td>Case 6: OK</td>
<td>Case 7: OK/KO</td>
<td>Case 8: OK/KO</td>
</tr>
</tbody>
</table>

Table 1: The height combinations of declarative conditionals and moods

Previous theories of the interaction between moods and conditionals cannot explain why only part of these cases are permissible. Indeed, their analyses are too coarse-grained to even represent these differences. For the theories of the first group in which a probabilistic treatment for indicative and a possible worlds semantics for counterfactuals are advocated, the cases in which the antecedent and the consequent differ in their respective mood is not even being considered. Furthermore, they have very slim prospects to be able to extend their theories to cope with these cases. Indeed, there is no reason to favor one semantics over another for the intermediate constructions. Finally, the analysis would need to be applied at the level of the antecedent and consequent, contrary to the level on which they actually operate which is the whole conditional.

With respect to the second difficulty, the theories of the second group which defend a unique semantics for the conditional and a modification of this semantics by the contribution of the moods are in a better shape. However, they are far from having a sufficient level of generality to actually deal with our present issue. Stalnaker (1975) defends that “an indicative conditional focuses solely on antecedent-worlds among the contextually live possibilities c, which represent what’s being taken for granted in the discourse” whereas “a subjunctive conditional focuses on antecedent-worlds that need not be among those possibilities.” The key point is that the moods are considered only relatively to the whole conditional and not relatively to its parts. The consequence is that a distinction is made only between indicative and subjunctive conditionals, without leaving any room to treat intermediate constructions. Finally, the effect of these moods only affects the way antecedent-worlds are interpreted, which means that the mood attached to the consequent has no impact on the whole semantics.

Subsequent theories have the same default. We can illustrate this claim with the two most recent ones. Starr (2014) reformulates Stalnaker’s distinction between the indicative and the subjunctive conditional inside a dynamic semantics. With ◦ the modal operator for the subjunctive, his formal representation

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2 See (Iatrídiou, 1994), (Geis and Lycan, 1993), (Lycan, 2001) or (Dancygier and Sweetser, 2005). Swanson provides a more complete list on this point.
of a subjunctive conditional is ‘(if A) C’, which means that his semantics modifies only the way the antecedent is interpreted. The mood used for the consequent in the natural language has therefore no counterpart at the formal level. Hence, he cannot distinguish between our cases 1 and 3, or between our cases 2 and 4. Khoo (2015) defends that the subjunctive carries an extra layer of past tense for the evaluation of the hypothetical sentence. However, this extra layer is applied on the whole conditional and not on its constituents. So, we have no way to interpret in his theory the intermediate constructions where the subjunctive/conditional mood is only applied to the antecedent or to the consequent.

In sum, all these theories keep the division between indicative and subjunctive as the only one possible for conditionals. They offer no formal tools allowing the formalization of other constructions. This is problematic because we showed that other combinations are possible in natural language and that the question of why some of them are forbidden is of great interest. Furthermore, these theories do not apply the compositional treatment observed in natural language, especially when the mood interacts with the consequent, and not with the whole conditional. To sum it up, despite all their virtues, these different theories are too narrowly conceived to deal with the interactions of moods and conditionals at a general level. The theory proposed in this paper will overcome these limitations.

3 Moods and Normal Conditionals

In this section, I will examine how normal conditionals and moods interact. To do this, I adopt the semantics presented in (Vidal, 2016, 2017) for conditionals. The reasons of this choice are twofold. Firstly, in this semantics, the different ways the realization of the antecedent is envisaged are delineated by what is called a universe of projection, a notion that will be useful for my analysis of the moods. Secondly, this theory offers the first compositional possible worlds semantics for if then and even if conditionals by combining the meaning of if, then and even. The conditional form presented here is the simple “if A, C” form. In this semantics which uses trivalent possible worlds, the judgment of a conditional is a two-stage process. First, the antecedent is inhibited: it is no more believed true nor false. Second, the antecedent is added again in several ways and the obtainment of the consequent is evaluated in these different reconstructions. The universe of projection can be understood as a set which restrains the possible reconstructions of the antecedent and its negation to the ones which are credible in the context of utterance. Finally, we add a further constraint to this primitive semantics. There always exists some situations where the antecedent is envisaged. This means that the set of antecedent-worlds cannot be null. The Figure 1 presents this semantics for the conditional “if A, C”, with w the starting world of evaluation, n the neutralization (inhibition) phase, e the expansion (addition) phase, and the square for the universe of projection.

Figure 1: Basic semantics of the conditional

More formally, the following truth-conditions are obtained for the sentence if A, C, that will be symbolized by A → C.

Theorem 3.1 (Truth-Conditions of a Normal Conditional).

∀w ∈ w, A → C iff in the associated universe of projection U

\[ w, A \rightarrow C \]
In order to model the moods, I will now add some formal constraints to this initial semantics by reusing the idea of context already put forth by (Stalnaker, 1975) and (Stalnaker, 1998). In this second article, Stalnaker explains us that he “propose[s] to identify a context (at a particular point in a discourse) with the body of information that is presumed, at that point, to be common to the participants in the discourse[...]. We can represent the information that defines the context in which a speech act takes place with a set of possible situations or possible worlds — the situations that are compatible with the information.” I take up his idea that the use of different moods in conditionals indicates a particular relationship with the context set. The indicative signals that the sentence is among the contextually live situations. By contrast, the subjunctive and conditional moods indicate that the sentence is not among the context set.

Despite all these hints, Stalnaker does not provide a complete formal analysis of the interactions between conditionals and context. The main difficulty of his theory is that it does not specify how these ideas can be applied to model the use of the mood restricted to the consequent. I will now provide such solution by expanding his proposal. First, the context will be a set of trivalent possible worlds noted $C$, which can evolve along the conversation. This represents what is taken for granted by everybody. This context can change but it is difficult to assert something against it without being ready to furnish explicit reasons. The subjunctive and the conditional moods are means to express incompatible information with this common background. By application of our previous general principle, we predict that a sentence with the conditional mood is difficult to assert alone and out of the blue. If someone says “Obama would not have been president” and that nothing in the previous discussion or in the previous background was pointing toward this surprising assertion, the hearers will probably ask explanations about this view before accepting such opinion. The speaker must therefore be ready to substantiate his claim. One way to introduce such sentence going against the context is to precede it with a hypothetical clause that gives a reason to the counterfactual happening of the consequent. This is generally the case for normal conditionals and especially for counterfactuals. For instance, the opening sentence of (Lewis, 1973) is “If kangaroos had no tails, they would topple over.” Alone, the consequent is not acceptable in the present context. However, preceded by the antecedent which introduces a hypothetical situation, a reason is now given for this counterfactual consequent.

Let us look at a formal presentation of these ideas. We first define the meaning of the moods.

**Definition 3.1. Meaning of the moods.** With a context $C$, a sentence $A$ and its truth-set $|A|$, the indicative sentence $A$ means that $|A| \subseteq C$ and the subjunctive/conditional sentence $\circ A$ means that $|A| \cap C = \emptyset$.

For the subjunctive/conditional case, some examples could be seen as directly contradicting this proposal, like Anderson-style sentences. An answer to these cases will be given after the complete exposition of the formal solution.

This definition can now be applied to the conditional and more precisely to its components. We start with the case of the antecedent. The protasis of a conditional is a relative clause and the set of possible worlds associated with it is clearly limited. Indeed, by asserting ‘if A’, we do not select all the worlds in which A is true but only those appearing to be the most plausible. By using the indicative, we mean that this particular set is among the context and by using the subjunctive, we mean that it is outside the context. In order to determine the exact truth-set of the antecedent, we must look at the truth-conditions given to the conditional. The first condition which states the non-emptiness of the result of the neutralization must be seen as a precondition. Hence, we must focus on the second condition $e_{n,(A)}(A) \subseteq [C]$ to find our answer. Clearly, the left part of this subset relation represents the truth-set of the antecedent. Notice that this set
is also used in the third condition. Hence, the meaning of the antecedent is the result of the expansion
function and not the totality of the A-worlds. Technically, for the antecedent A, \( |A| = e_{n(A)}(A) \). We
therefore obtain the following meaning for the compositional application of the moods to an antecedent.

**Theorem 3.2.** Indicative and subjunctive moods for an antecedent. The indicative antecedent A means
that \( e_{n(A)}(A) \subseteq C \). The subjunctive antecedent \( \circ A \) means that \( e_{n(A)}(A) \cap C = \emptyset \).

The indicative and the subjunctive cases entail that the result of the expansion function is respectively
inside and outside the context set. This result is similar to Stalnaker’s proposal which is formulated with
the help of a selection function.

I shall now examine the case of the consequent. Grammatically, it is a main clause, but we cannot
adopt the idea that the set of all the worlds where the consequent is true is inside or outside the context set.
For a consequent C, this would mean that all C-worlds are included or excluded from C. This represents
too much unrelated possibilities to what the speaker has in mind. Hence, we need to find a way to restrict
our set to the only possibilities that the speaker contemplated. The notion of universe of projection,
which is already present in the semantics, will be used for this. Indeed, it brings together all the possible
reconstructions for the antecedent and the consequent of the conditional at hand. More formally, by
examining the truth-conditions of the conditional \( A \rightarrow C \), only the second condition \( e_{n(A)}(A) \subseteq [C] \) is
of interest in order to determine the truth-set of the consequent. We see that this truth-set must be the
right part of the subset relation, that is the set of C-worlds. But in this relation, this set is restricted to the
universe of projection \( U \). Hence, for a consequent C, \( |C| = U \cap [C] \). We can now compositionally derive
the meaning resulting from the application of moods to a consequent.

**Theorem 3.3.** Indicative and conditional moods for a consequent. Let \( U \) be the universe of projection
of the conditional. The indicative consequent C means that \( (U \cap [C]) \subseteq C \). The conditional consequent
\( \circ C \) means that \( (U \cap [C]) \cap C = \emptyset \).

Hence, with the indicative mood, the set of consequent-worlds restricted to the universe of projection
is compatible with the context. With the conditional mood, this set is incompatible.

Let us now look at the different combinations of moods with normal conditionals, in order to examine
whether a true conditional is compatible with the additional formal constraints provided by the moods. As
the conditional is true, the condition \( n_{\circ}(A) \neq \emptyset \) is obtained in all the following cases and will be set aside
in the following explanations because it has no influence on the possible or impossible combinations.

**Case 1:** \( A \rightarrow C \)
\[ e_{n(A)}(A) \subseteq [C], e_{n(A)}(A) \neq \emptyset, e_{n(A)}(A) \subseteq C \text{ and } (U \cap [C]) \subseteq C. \text{ No contradiction.} \]

**Case 2:** \( \circ A \rightarrow C \)
\[ e_{n(A)}(A) \subseteq [C], e_{n(A)}(A) \neq \emptyset, e_{n(A)}(A) \cap C = \emptyset \text{ and } (U \cap [C]) \subseteq C. \text{ Contradiction because } e_{n(A)}(A) \subseteq [C] \text{ is equivalent with } e_{n(A)}(A) \subseteq (U \cap [C]). \text{ Thus, } e_{n(A)}(A) \subseteq C. \text{ This contradicts } e_{n(A)}(A) \cap C = \emptyset \text{ because } e_{n(A)}(A) \neq \emptyset. \]

**Case 3:** \( A \rightarrow \circ C \)
\[ e_{n(A)}(A) \subseteq [C], e_{n(A)}(A) \neq \emptyset, e_{n(A)}(A) \subseteq C \text{ and } (U \cap [C]) \cap C = \emptyset. \text{ Contradiction because } e_{n(A)}(A) \neq \emptyset. \]

**Case 4:** \( \circ A \rightarrow \circ C \)
\[ e_{n(A)}(A) \subseteq [C], e_{n(A)}(A) \neq \emptyset, e_{n(A)}(A) \cap C = \emptyset \text{ and } (U \cap [C]) \cap C = \emptyset. \text{ No contradiction.} \]

Compared to our initial data, we obtain the desired results: only the cases 1 and 4 are possible, the
other ones are forbidden.

Actually, we limited our inquiry to conditionals without modals. But an in-depth study of such mix-
tures would be interesting. For instance, the combination 3 can receive a formulation which is now
completely correct. Kid: “I’m hungry”, Parent: “if you are hungry, you should have said it before.” This last sentence expresses a general rule and the conditional mood indicates that the rule does not hold in this particular situation. This paper is not the place to solve the issue of how modals and conditionals precisely combine. But the correct conditions that seem desirable in order to treat this case can at least be provided. As usual, the modal operator is symbolized by the square \( \Box \) and the idealized worlds where the obligation \( C \) is fulfilled are represented semantically by ‘I(C)’.

**Case 3 revisited:**

\[
e_{m_1}(A) \subseteq [I(C)], e_{m_2}(A) \neq \emptyset, e_{m_3}(A) \subseteq C \text{ and } (U \cap [C]) \cap C = \emptyset. \text{ No more contradiction.}
\]

Notice that this type of sentence does not only concern moral or legal statements but all conditionals expressing an obligation. For instance, the following example depicts a meteorological relation: “If it was sunny this morning, it should have been no rain.” Hence, this notion of obligation is not restricted to deontic notions.

The idea that the context set varies with the conversation and can be revised is illustrated by the famous example given in (Anderson, 1951) against the idea that all subjunctive conditionals are counterfactuals:

(18) If Jones had taken arsenic, he would have shown just exactly those symptoms which he does in fact show.

This sentence is asserted in a context where arsenic poisoning is not previously accepted by the participants in the conversation. If we symbolize (18) by \( \Diamond A \rightarrow \Diamond S \land S \), we have the following conflicting beliefs to accommodate:

**Analysis of (18):**

\[
e_{m_1}(A) \subseteq (U \cap [S]), e_{m_2}(A) \neq \emptyset, e_{m_3}(A) \cap C = \emptyset, (U \cap [S]) \cap C = \emptyset \text{ and } (U \cap [S]) \subseteq C. \text{ Contradiction because } e_{m_3}(A) \neq \emptyset.
\]

The solution proposed to this puzzle is the following one. We are able to imagine that Jones shows the symptoms that we actually observe. Hence, \( e_{m_3}(A) \) and by consequence \( (U \cap [S]) \) are certainly not empty. This conducts to a context set which is contradictory and which must be revised. In this case, the last information carried is kept (“he shows the symptoms of arsenic”) and its opposite is removed (“he would have shown the symptoms of arsenic”). We obtain now:

**Elimination of the first contradiction**

\[
e_{m_1}(A) \subseteq (U \cap [S]), e_{m_2}(A) \neq \emptyset, e_{m_3}(A) \cap C = \emptyset \text{ and } (U \cap [S]) \subseteq C. \text{ No more contradiction.}
\]

Now, the antecedent-worlds are both outside and inside the context set. Two cases are possible. Either the hearer stands with the refusal of the arsenic ingestion for the present context, for instance if he has toxicological analysis stated by a forensic doctor. In that case, the revision process does not go further. Or the first revision and the consideration that the victim actually shows the arsenic symptoms are sufficiently strong to consider that this type of poisoning is not so far-fetched. In that case, the revision continues and to solve the contradiction, the oldest information which is the impossibility of arsenic poisoning in the context is removed:

**Elimination of the second contradiction**

\[
e_{m_1}(A) \subseteq (U \cap [S]), e_{m_2}(A) \neq \emptyset \text{ and } (U \cap [S]) \subseteq C. \text{ No more contradiction.}
\]

After this second revision, the antecedent-worlds being a subset of the consequent-worlds, they are also in the context. Hence, we can conclude by an abductive reasoning that Jones’s disease is caused by an ab-
sorption of arsenic. Anderson’s example illustrates how the context set is a revisable notion. This process of revision is the following: in presence of contradictory data of form $A \land \neg A$, if the revision is worth to be carried, keep the most recent and certain information and remove the oldest or the less reliable one. In the case of Anderson’s example, the contradiction is worth to be resolved because the speaker deliberately asserts the consequent with two different moods. The hearer infers that this is surely done with purpose and tries to give it a sense. The revision is valuable because it conducts to establish arsenic poisoning, something which seems impossible at the beginning. Finally, let us remark that the solution presented here to Anderson-style examples uses the application of the meaning of the mood on the consequent, an aspect which is absent of preceding theories. This is certainly why they are unable to precisely explain this type of sentences.

4 Moods and Relevant conditionals

A relevant conditional expresses a relation of optimality between its antecedent and its consequent. This type is also called “nonconditional conditional” in (Geis and Lycan, 1993), “speech-act conditional” in (Dancygier and Sweetser, 2005) and “biscuit conditional” from the famous example offered in (Austin, 1961) and that we reuse here with slight modifications.

(19) If you are hungry, there are biscuits on the table.

Here, the assertion of the conditional clause makes the assertion of the main clause more acceptable in the context of the discourse. The information given by the consequent is more apposite in the context settled by the antecedent. The function of the antecedent is to increase the relevance of the information in the consequent, comparatively to the previous topic in the discourse. In this example, the location of the biscuits is of interest only in a context where the hearer is hungry. Instead of giving this information out of the blue, the speaker prefers to set up a context where it will be welcome. Hence, in this type of conditional, the realization of the consequent is independent of the realization of the antecedent. But the information carried by the apodosis is made more relevant in the frame of reference set by the protasis. The assertion of the antecedent allows a change of context, which in turns secures optimality of the utterance of the consequent.

Depending on the circumstances, this optimality concerns different aspects. The preceding sentence gives an implicit authorization to the hearer — he can eat the biscuits. Another objective of the speaker could be to respect some norms of social conduct. For instance, the following example proposed by (Noh, 1998) shows that the antecedent could excuse a non-respect of the rules of communications, like an improper use of language:

(20) He trapped two mongeese, if that’s how you make the plural of “mongoose”.

In the same way, a speaker must comply with norms of politeness. In particular, an unpleasant remark could be considered to be an insult. The hypothetical form allows lessening the strength of the main clause, as shown by this example from Franke (2007):

(21) If I may say so, you are not looking good.

In the same vein, the antecedent can underline that the speaker acts against the will of the hearer:

(22) If you don’t want to know the truth, he’s lying.

Finally, a relevant conditional can also make clearer the introduction of the consequent in the general progression of the discourse. For instance, the following sentence could be uttered during a presentation:

(23) If we can now address our last issue, the situation is bad.
Hence, relevant and normal conditionals slightly differ in meaning. For the first ones, the consequent is more acceptable if the antecedent is true. For the second ones, the consequent is true if the antecedent is true. But we reach here a limit for a pure compositional approach to the formal semantics of conditionals. From a syntactic point of view, a relevance conditional does not introduce any variation compared to a normal conditional — we obtain a sentence of form ‘if A, C’ in both cases. To discriminate between the two types, we need first to understand that the relevance conditional expresses a relation of optimality and that normal conditional expresses a relation of dependence between the realizations of the facts. This difference of meaning cannot be detected by a variation in syntax. However, differences arise when we consider the enrichment of the conditional connective with additional markers, like the word ‘only’:

(24) Only if you are hungry, there are biscuits on the table.

In (24), the speaker asserts that the presence of the biscuits on the table depends on the hungriness of the hearer. Here, we do not face anymore a relevant conditional. Two options can explain this phenomenon. Either the word ‘only’ transforms the initial relevant meaning into a normal one. Or the word ‘only’ is incompatible with the relevant interpretation and forces a normal interpretation. Whatever is the right explanation, this shows that a relevant conditional deserves its own particular semantics.

Let us now turn to a formal exposition of its meaning. Following (Iatridou, 1991), I argue that “the if-clauses in biscuit conditionals specify the circumstances in which the consequent is relevant (in a vague sense, also subsuming circumstances of social appropriateness), not the circumstances in which it is true.” Hence, the antecedent does not lead to the circumstances in which the consequent is true but to the circumstances in which it is relevant. Let us symbolize the set of possible worlds where C is relevant with \( \mathcal{R}(C) \). Then, we obtain the following truth-conditions for the relevant conditional, that we represent with the connective \( \rightarrow^* \).

**Theorem 4.1 (Truth-Conditions of a Relevant Conditional).**

\[
\vDash_w A \rightarrow^* C \text{ iff in the associated universe of projection U}
\]

i) \( n_w(A) \neq \emptyset \)

ii) \( e_{n_w(A)}(A) \subseteq \mathcal{R}(C) \)

iii) \( e_{n_w(A)}(A) \neq \emptyset \)

Unsurprisingly, the definitions of normal and relevant conditionals are very close. They only differ concerning the set of possible worlds denoted by the consequent. However, this change has an important consequence. For a relevant conditional, the facts denoted by the antecedent and the consequent happen independently. Furthermore, as the consequent is expressed in a main clause, its realization is directly asserted, contrary to the antecedent which is hypothetically introduced by the particle ‘if’. This contradicts the view of DeRose and Grandy (1999) who consider this type of sentence as a conditional assertion of the consequent — if the antecedent is false, nothing is asserted. But as already showed by Ebert et al. (2008), the assertion and the speech act associated with the consequent are always undertaken, whatever is the truth-value of the antecedent. The following example clearly shows this point:

(25) If you don’t want to see the movie, the murder is the aunt.

Even if the speaker is mistaken regarding the intentions of the hearer (meaning that the antecedent is false), the damage is done: the hearer knows the culprit as soon as the sentence is asserted.

Let us now turn to the combination of moods with relevant conditionals. We saw that the notion of context set restrains what is assertable in the discourse. In particular, a consequent, which is a main clause, cannot go against the context set if its antecedent does not give a reason for this incompatibility. In the case of a relevant conditional, the antecedent does not explain the realization of the consequent but only increases its acceptability, compared to the norms of conversation. Thus, a consequent with the
conditional mood is not permissible in an isolated relevant conditional, because the antecedent does not explain its realization. However, if other clarifications previously provided in the discourse can explain the incompatibility of the consequent with the context set, a relevant reading of the conditional is acceptable with the conditional mood. Hence, an isolated relevant conditional cannot contain a consequent in the conditional mood, contrary to a relevant conditional in a discourse. This limitation is expressed by the following rule.

Definition 4.1. Non-assertion of a relevant conditional with a conditional consequent. A consequent at the conditional mood cannot be asserted inside a relevant conditional if no other piece of information can explain this release of the context set.

We can now examine the four uses of moods in relevant conditionals:

Case 5: $A \rightarrow^* C$
$\epsilon_{m(A)}(A) \subseteq R(C), \epsilon_{m(A)}(A) \neq \emptyset, \epsilon_{m(A)}(A) \subseteq \emptyset \cap (U \cap [C]) \subseteq \emptyset$. No contradiction.

Case 6: $\circ A \rightarrow^* C$
$\epsilon_{m(A)}(A) \subseteq R(C), \epsilon_{m(A)}(A) \neq \emptyset, \epsilon_{m(A)}(A) \cap \emptyset = \emptyset \cap (U \cap [C]) \subseteq \emptyset$. No contradiction.

Case 7: $A \rightarrow^* \circ C$
$\epsilon_{m(A)}(A) \subseteq R(C), \epsilon_{m(A)}(A) \neq \emptyset, \epsilon_{m(A)}(A) \subseteq \emptyset \cap (U \cap [C]) \cap \emptyset = \emptyset$. A conditional consequent is not assertable inside a relevant conditional if no further explanation is given in the discourse. This accounts for the difference of acceptability between the examples (12) and (14).

Case 8: $\circ A \rightarrow^* \circ C$
$\epsilon_{m(A)}(A) \subseteq R(C), \epsilon_{m(A)}(A) \neq \emptyset, \epsilon_{m(A)}(A) \cap \emptyset = \emptyset \cap (U \cap [C]) \cap \emptyset = \emptyset$. Again, a conditional consequent is not assertable inside a relevant conditional if no further explanation is given in the discourse. This accounts for the difference of acceptability between the examples (13) and (15).

Finally, the results obtained are consistent with the empirical data. The cases 5 and 6 are totally licit. The cases 7 and 8 are not permissible used in isolation, but possible with some previous explanations allowing the use of the conditional mood.

5 Application to three puzzles

The theory advanced in this paper can be seen as an extension of Stalnaker’s proposal, operating at a more general level. Its main advantage is that it explains what are the possible combinations of moods and conditionals in natural language sentences. In this section, I will examine how this formalization of the moods in conditionals allows tackling three well-known problems: the Oswald-Kennedy example, the direct argument and the Gibbardian stand-offs. I show here that these three issues can be solved in a unified way. More precisely, in each case, the solution of these problems lies on the notion of context set that was previously exposed. Furthermore, depending on the puzzles examined, this solution is equivalent or better than the one provided in a standard Stalnakerian treatment.

5.1 Oswald and Kennedy

Famously, Adams (1970) proposed the following example in order to show a major difference between indicative and subjunctive conditionals:

(26) If Oswald did not kill Kennedy, then someone else did.
(27) If Oswald had not killed Kennedy, then someone else would have.

Sentence (26) is certainly true, in contrast to sentence (27). However, the only difference between these two sentences is the mood used. The conclusion taken by many authors is that this difference of truth-value shows that the indicative and subjunctive moods must be modeled through a completely different semantics. I will argue against this view by showing that this difference of truth-value can be explained by the previous analysis of the interactions between conditionals and moods.

First, let us notice that this difference induced by the mood is not particular to conditionals. This is shown by the pair of following sentences:

(28) Without his visit, I finished my work on time.
(29) Without his visit, I would have finished my work on time.

In sentence (28), no visitor disturbs the speaker’s work, in contrast to (29). The first sentence describes a real fact and the second sentence a counterfactual situation. Both sentences differ in meaning and this shift from a realistic context to a hypothetical one is due to a modification of the mood. We do not see why conditionals should take place in the explanation of the present case, because no conditional is used in these two sentences. This difference of meaning is certainly more easily explainable from a theory which emphasizes the interaction between the moods and the context set and which is more general. The natural explanation given in the present framework is the following one. By symbolizing the sentence “I finished my work on time” by \( F \) and its truth-set restricted to this context of assertion by \( [F] \), we can conclude that \( [F] \subseteq C \) for the first case and \( [F] \cap C = \emptyset \) for the second case.

Let us now turn to the formal examination of the Oswald-Kennedy case. Sentences (26) and (27) will be represented by ‘\( \neg A \rightarrow C \)’ and ‘\( \neg A \rightarrow C \)’. We are able to judge both sentences and to envisage the realization of the antecedent so the inhibition of the antecedent is possible and antecedent-worlds are attained. Hence, we have the conditions \( n_w(\neg A) \neq \emptyset \) and \( e_n(\neg A) \neq \emptyset \) for free. Furthermore, everybody evaluating these two conditionals believes that either Oswald or someone else killed Kennedy because we know that the president was murdered. Hence, \( A \vee C \) is a common background assumption. Finally, we use a further semantic condition that we did not yet present: for all sentence \( A \), \( e_n(\neg A) \subseteq [A] \).

This means trivially that all the worlds obtained by reconstructing the antecedent are worlds where this antecedent is true. In conditional logic, this condition is adopted by all systems which are considered sufficiently strong to represent our real use of hypothetical sentences. It is called (id) in (Chellas, 1975), because it validates identity.

**Theorem 5.1.** “If Oswald did not kill Kennedy, then someone else did” is true with the preceding assumptions.

**Proof.**

1. \( n_w(\neg A) \neq \emptyset \) and \( e_n(\neg A) \neq \emptyset \) (background assumption)
2. \( C \subseteq [A \vee C] \) (background assumption)
3. \( e_n(\neg A) \subseteq C \) (indicative mood)
4. \( e_n(\neg A) \subseteq [A \vee C] \) (from 2. and 3.)
5. \( e_n(\neg A) \subseteq [\neg A] \) (id)
6. \( e_n(\neg A) \subseteq [C] \) (from 4. and 5.)
7. \( \neg A \rightarrow C \) (from 1. and 6.)

This demonstration cannot be carried for the subjunctive case because we do not have anymore the condition linked to the indicative mood. Formally, the validity of ‘\( \neg A \rightarrow C \)’ can receive a counter-model.
Theorem 5.2. “If Oswald had not killed Kennedy, then someone else would have” can be false with the preceding assumptions.

Here, we present a counter-model with the Figure 2. As the antecedent is fictional, $e_{n(\neg A)}(\neg A)$ is outside the context set. Thus, no constraint coming from the background assumptions applies to worlds outside $C$. Therefore, we can very well imagine that at least one world in $e_{n(\neg A)}(\neg A)$ makes the consequent false: no one else than Oswald intended to kill Kennedy.

![Figure 2: Counter-model: no one else killed Kennedy](image)

The difference of truth-values between these two sentences can be explained by the additional semantic constraints coming from their moods. Furthermore, the underlying semantics for the conditional operator is the same in both cases. This answer is very similar to the one given by Stalnaker. Indeed, for him, the difference between the two types of conditionals comes from the fact that the range of the selection function is restricted by the context set in the case of indicatives, but not in the case of subjunctives. In the present theory, the range of both the expansion function and the truth-set representing the consequent lies inside the context set for indicatives, but not for subjunctives.

5.2 Direct Argument

The puzzle introduced by (Stalnaker, 1975) shows the closeness of disjunction and conditional. Imagine the following situation. A detective, dealing with a criminal case, manages to reduce the number of suspects to two persons: the butler and the gardener. Then, he can rightly assert:

(30) Either the butler or the gardener did it. Therefore, if the butler didn’t do it, the gardener did.

This piece of reasoning seems correct. We thus obtain the following inference:

(31) (Direct Argument) $A \lor C \models \neg A \rightarrow C$

However, as $A \lor C \equiv \neg A \supset C$, we also obtain that from the material conditional, we can systematically derive the indicative conditional:

(32) (Material to indicative conditional with negation) $\neg A \supset C \models \neg A \rightarrow C$

(33) (Material to indicative conditional) $A \supset C \models A \rightarrow C$

Furthermore, many authors consider the opposite inference as valid: each time an indicative conditional is true, the corresponding material conditional is also true. The devastating conclusion is that the indicative conditional is simply the material conditional, carrying with it all its paradoxes. This argument has been set forth, for instance by Edgington (1995) in order to defend the view that ‘if’ is not a truth-functional operator. The direct argument is the main ground for this position, named NTV for ‘No Truth Value’ in (Bennett, 2003). Tenants of this conception deny that one could ascribe truth-values to indicative conditionals and promote probabilities as the correct tool to analyze their assertability.

Let us first expose Stalnaker’s solution to this puzzle and its difficulties. Without presenting in details all the notions entering in his proposal, it can be sum up in the following way.

1. $A \lor C$ is acceptable in the context set (hypothesis)
2. \( \neg A \land C \) is compatible with the context (from 1., disjunction and the notion of compatibility)
3. \( \neg A \) is compatible with the context (from 2., and the notion of compatibility)
4. \( \neg A \rightarrow A \lor C \) is acceptable in the context set (from 1., 3. and the pragmatic constraints)
5. \( \neg A \rightarrow C \) is acceptable in the context set (from 4. and the validity of the derivation)

This solution presents a major problem. To obtain the fourth line of this reasoning, Stalnaker uses the following pragmatic constraint: “if a proposition \( P \) is compatible with the context, and another proposition \( Q \) is accepted in it, or entailed by it, then the conditional, if \( P \), then \( Q \), is entailed by it as well.” The problem of this constraint is that it allows the entailment of conditionals without any relevance between their antecedent and consequent. For instance, the butler committed the crime is compatible with the context. Furthermore, green is a color is accepted in the context. Therefore, if the butler committed the crime, green is a color is accepted in the context. However, several psychological experiments (Matalon, 1962; Skovgaard-Olsen et al., 2016; Vidal and Baratgin, 2017) showed that conditionals without any link between the antecedent and the consequent have no clear acceptance by the participants. For instance, the sentence “if 1 + 1 = 2, Pacific is an ocean” is far from being considered true by everybody, despite the truth of its elements. Some could defend Stalnaker’s position by saying that the notion of acceptance used here is not the common one but a technical one framed especially to model conversation. The problem of this defense is that Stalnaker gives us no further hint on what could be this technical definition. Hence, ‘an assertion accepted in a context’ must be taken at face value and with this current form, the pragmatic constraint proposed by Stalnaker entails the acceptance of irrelevant conditionals.

I will now show that another solution based on the notion of context set and free from these objections can be found. Indeed, the derivation of the indicative conditional from the disjunction is possible if we add a few background assumptions. First, we are able to imagine situations in which the butler did not commit the crime. Hence, we can freely inhibit the antecedent and consider antecedent worlds: \( n_u(\neg A) \neq \emptyset \) and \( e_{n_u(\neg A)}(\neg A) \neq \emptyset \). Second, the fact that the crime was committed by the butler or the gardener is well-established. Hence, we can transform the original premise into the following one: \( C \subseteq [A \lor C] \).

**Theorem 5.3.** With these preceding assumptions, we can derive the truth of \( \neg A \rightarrow C \).

**Proof.**
1. \( n_u(\neg A) \neq \emptyset \) and \( e_{n_u(\neg A)}(\neg A) \neq \emptyset \) (background assumption)
2. \( C \subseteq [A \lor C] \) (background assumption)
3. \( e_{n_u(\neg A)}(\neg A) \subseteq C \) (indicative mood)
4. \( e_{n_u(\neg A)}(\neg A) \subseteq [A \lor C] \) (from 2. and 3.)
5. \( e_{n_u(\neg A)}(\neg A) \subseteq [\neg A] \) (id)
6. \( e_{n_u(\neg A)}(\neg A) \subseteq [C] \) (from 4. and 5.)
7. \( \neg A \rightarrow C \) (from 1. and 6.)

This proof explains the inference from disjunction to conditional for this particular example. Notice that this proof is exactly the same as the one used for the indicative case of the Oswald-Kennedy example. Indeed, both puzzles exhibit the same background assumptions. In particular, a disjunctive premise is true for the whole context set. The only difference between the two examples is that in the Oswald-Kennedy case, this disjunction is implicit, contrary to the present case where the disjunction is explicit. This is a major strength of the theory offered here to unify the solution to these two problems.

The direct argument offers a case where the schema (30) is true. However, for this argument to be really convincing, the derivation from disjunction to indicative conditional would need to be valid, that is true under all circumstances. This means also the validity of the following inference, which is only a variation of (30) concerning the place of the negation:
Contrary to what is suggested by the butler-gardener example, this inference is far from being true in all cases and it is easy to find counterexamples. For instance, imagine that the sentence “it does not rain outside”, symbolized by \( \neg A \), is actually true. From the semantics of the disjunction, we can conclude that both \( \neg A \lor C \) and \( \neg A \lor \neg C \) are true. Thus, by (34), both \( A \rightarrow C \) and \( A \rightarrow \neg C \) are true. Thus, the validity of this inference would entail that one believe simultaneously that “if it rains outside, I take my umbrella” and “if it rains outside, I do not take my umbrella”, which seems impossible. Hence, this inference is not always true and the equivalence between the material conditional and the indicative conditional from natural language cannot be sustained in all cases. The direct argument is just a particular example where this derivation is possible, but it does not entail that this equivalence is always valid.

5.3 Gibbardian Stand-Offs

Another important argument in favor of the NTV position is offered by Gibbard (1981), who details a situation where two persons believe in justified but opposite conditionals. Gibbard argues that this case undermines truth-conditional theories of conditionals because these theories must support the principle of conditional non-contradiction (CNC): \( \neg [(A \rightarrow C) \land (A \rightarrow \neg C)] \). We just saw an application of this principle by saying that it is not possible to believe simultaneously “if it rains outside, I take my umbrella” and “if it rains outside, I do not take my umbrella”. Bennett (2003) reformulates this puzzle to prevent any subjunctive reading of the conditionals. We will examine Bennett’s version of the argument here.

A system of three water-gates allows the water to run westwards or eastwards and is constrained by the following rules:

(35) If west and east gates are raised, the top gate is closed.

(36) If top and west gates are raised, the water runs westwards.

(37) If top and east gates are raised, the water runs eastwards.

Wesla is near the west gate and Esther is near the east gate. They are out of sight of each other and cannot communicate among themselves. Their task is to tell me what they see. The situation is pictured in Figure 3.

![Figure 3: Gate System](image)

Imagine now that west gate and east gate are raised. Then, each of my informants can tell me by phone:
The issue raised by Gibbard and Bennett is that these two sentences seem contradictory. Furthermore, if both are true, the CNC cannot be true. The first thing to notice is that the CNC is the well-known AB' thesis or Abelard’s First Principle of connexive logic. The validity of this principle is sometimes questionable, in particular when the formulas involved are tautologies or contradictions. But admittedly, when it concerns everyday reasoning, we can suppose that it is valid, as what we done for the direct argument. Thus, sentences (38) and (39) cannot be both true. But which one is the right one to choose, if there is any?

Let us now turn to the solution of Gibbard’s phenomenon in the present framework. First, the two conditionals are uttered by two different persons with different information on the situation. Hence, in their own set of beliefs, their utterance does not entail any contradiction. Furthermore, they do not share a context set, having no common conversation. The problem occurs when the hearer tries to integrate both sentences in its own context set. I will show that the only way to reconcile both data is to conclude that no possible world can be obtained for the antecedent. We use the proposition T for “the top gate is raised”, W for “water will flow westwards” and E for “water will flow eastwards.”

**Proof.**

1. \( T \rightarrow (W \land \neg E) \) (Wesla’s information)
2. \( T \rightarrow (E \land \neg W) \) (Esther’s information)
3. \( n_e(T) \neq \emptyset, e_{n_e(T)}(T) \subseteq \emptyset, e_{n_e(T)}(T) \subseteq [W] \cap \neg [E] \) (from 1)
4. \( n_e(T) \neq \emptyset, e_{n_e(T)}(T) \neq \emptyset, e_{n_e(T)}(T) \subseteq \emptyset, e_{n_e(T)}(T) \subseteq [E] \cap \neg [W] \) (from 2)
5. Contradiction because \( e_{n_e(T)}(T) \neq \emptyset \) (from 3. and 4) \( \square \)

The conclusion is that the hearer cannot imagine a situation where the antecedent is realized without obtaining a contradiction. Therefore, the top gate cannot be raised and this is the right conclusion to make. Hence, the hearer can make the correct prediction, if he follows the formal semantics defended here. This is an important advantage of the present theory. This also means that there is no possible world which can allow the truth of these opposite consequents. Hence, both conditionals cannot be accepted by the hearer, because both of them have an antecedent which cannot be made true in the situation at hand. Thus, this solution has the second advantage to not contradict the CNC.

Williams (2008) already offered an explanation of Gibbard’s phenomenon based on Stalnaker’s context set. However, his approach presents important differences with what is defended here. Indeed, he concludes that “[his] account of the truth-conditions of indicative conditionals […] allow[s] for both […] utterances to both be true in a single context.” In the opposite, the present proposal does not conclude that from the falsity of the antecedent, both conditionals will be considered true by the hearer. This seems psychologically unsustainable and contrary to the CNC. On the contrary, the present theory predicts that both conditionals will be false.

An objection to the present solution could be that in the Gibbardian stand-offs, it is a standard assumption that Wesla and Esther’s utterances are true. Predicting that both conditionals are false, this solution would not answer to Gibbard’s puzzle but to another one which would be how can we learn from false testimony. This objection does not stand up to analysis. Imaging that we have an observer in a helicopter who can see the whole configuration of the gates. He knows that the top gate is closed and that the other two gates are raised. He cannot therefore assert Wesla and Esther’s conditionals, because their antecedent is false. So, from an objective point of view, both conditionals are false. Hence, if it is

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(38) Wesla: “If the top gate is raised, water will flow westwards, not eastwards”

(39) Esther: “If the top gate is raised, water will flow eastwards, not westwards”

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See (Priest, 1999) and (Wansing, 2010) for an introduction.
a standard assumption that both conditionals are objectively true in the Gibbardian stand-offs, then the puzzle, in its present form and in its original form is poorly built. It seems preferable to consider that the puzzle is well-presented and that Wesla and Esther believe that their utterances are true, even if they are wrong on this point.

This leads us to consider the general issue settled by Gibbard’s puzzle: can conditionals have objective truth-values? We saw that both speakers can utter opposite conditionals because each one has its own set of information. Hence, conditionals are asserted based on epistemic considerations, which are by definition subjective. However, if a person knows all the relevant facts concerning the situation at hand, she will be able to give objective truth-conditions for the conditional judged. And this is what is done by the hearer in the present solution.

As a final point, let us make an assessment of the theory presented in this paper. Concerning our three last puzzles, the solutions offered are equivalent or better than Stalnaker’s orthodoxy. For the Oswald and Kennedy example, the predictions are sensibly the same. By contrast, the result is better in the case of the direct argument, because no use is made of the deficient pragmatic constraint on acceptance to deduce the conditional from the disjunction. The solution is also far better in the case of the Gibbardian stand-offs because firstly no contradiction is obtained between the conditionals and secondly their deduced truth-values are consistent with the truth-values that an objective observer would give to them. As an aside, it also gives an answer to Anderson-style examples. Finally, being more general and being able to predict all the possible combinations of moods and conditionals in natural language, the present theory offers strong advantages compared to the previous ones.

References


